

# LIKE A ROLLING BALL

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ABSTRACT. In this section we show two things: a) that the curve of the velocity of a rolling ball has three well defined segments corresponding to two phase transitions, and b) that a robot pushing a ball at a constant velocity can keep it “glued” to the robot chassis by just reducing slightly the rotational velocity of the ball. When robot and ball move at the same speed  $v$ , anytime the robot loses contact with the ball, the speed of the ball falls rapidly below  $v$ . The robot makes then contact with the ball again.

## 1. ROLLING BALL DYNAMICS

A ball can slip on a surface, can roll, or both. A ball slipping on a surface is decelerated by the friction force opposing the ball linear displacement. The friction force provides also angular momentum to the ball, which starts rotating until it rolls on the surface without slipping. When the ball is slipping, we have kinetic friction between the ball and the surface; when it is rolling we have rolling friction between the ball and the surface. The coefficients  $\mu_r$  and  $\mu_k$  of rolling and kinetic friction, respectively, are different. Kinetic friction depends on the weight of the object being pushed. Rolling friction depends on the deformation of the ball and the deformation of the floor – it is very difficult to handle theoretically and can only be assessed experimentally.

When a ball of mass  $m$  is slipping, the kinetic friction force  $F_k$  is proportional to the weight of the ball, as given by the formula

$$F_k = \mu_k mg.$$

The kinetic friction coefficient (and thus the force) is independent of the velocity of the ball, within certain limits (some cm/s up to several m/s).

A force applied to an object modifies its linear and angular momentum. The kinetic friction force  $F_k$  acts on the ball according to Newton’s second law

$$F_k = m\dot{v}$$

Where  $v$  is the velocity of the ball, and  $\dot{v}$  its first derivative. The angular momentum is modified according to the formula

$$F_k R = I\dot{\omega}$$

where  $R$  is the radius of the ball,  $\omega$  the angular velocity,  $\dot{\omega}$  its first derivative, and  $I$  the moment of inertia of the ball. For a ball of mass  $m$  and radius  $R$  the moment of inertia is  $\frac{2}{5}mR^2$ . Therefore

$$F_k = \frac{2}{5}mR\dot{\omega}$$

Noting that negative changes of  $v$  correspond to positive changes of  $\omega$ , we can equate the above formulas to obtain

$$m\dot{v} = -\frac{2}{5}mR\dot{\omega}$$

Therefore

$$\dot{v} = -\frac{2}{5}R\dot{\omega}$$

This is a “conservation of momentum” expression. The negative change in linear momentum of the ball is balanced by a positive change in the angular momentum of the ball. It is interesting to note that the relationship between  $\dot{v}$  and  $\dot{\omega}$  is independent of the mass of the ball. Integrating this expression for a ball that starts with a velocity  $v_i$  and angular velocity  $\omega_i$  and ends with velocity  $v_f$  and angular velocity  $\omega_f$ , we obtain

$$v_f - v_i = -\frac{2}{5}R(\omega_f - \omega_i).$$

One interesting case to consider is when the ball starts moving with slippage at velocity  $v_0$  and no rotation. When the ball reaches a state of rolling without slipping, the rolling velocity  $v_r$  is related to the rolling angular velocity  $\omega_r$  by  $v_r = \omega_r R$  and therefore we have

$$v_r - v_0 = -\frac{2}{5}v_r$$

From this we obtain  $v_r = \frac{5}{7}v_0$ . This means, that the rolling velocity is always five sevenths of the original velocity regardless of mass and radius of the ball.

## 2. ROLLING BALL VELOCITY CURVE

In this section we consider the form of the velocity curve for a ball which starts slipping on the surface and then rolls.

The kinetic friction force is given by

$$F_k = \mu_k mg$$

When the ball is slipping, the change in velocity is given by

$$v' = \mu_k g$$

This means that the change in the speed of the ball is linear.

The ball starts rolling without slipping when the velocity reaches  $5/7$  of the start value. Then, the ball moves theoretically without friction. However, a golf ball (such as the one used in RoboCup) is not exactly round. Also, the surface is a carpet, so that the ball sinks and has to climb over the next section of carpet. If the ball rolls too slowly, it cannot “climb” the carpet and stops moving. This is called rolling friction.

Figure 1 shows the curve of the ball speed for a slow kick. One can see clearly that at the beginning the ball is slipping and the change in velocity is linear and large, due to kinetic friction. When the ball starts rolling, kinetic friction is suddenly cancelled and rolling friction produces the deceleration of the ball. A perfect ball, on a perfectly flat surface, would roll much longer. This ball is a golf ball, that is, it is not completely spherical and the carpet is not perfectly flat. One can also see that the knee of the curve is located at around 4 cm/s which approximately

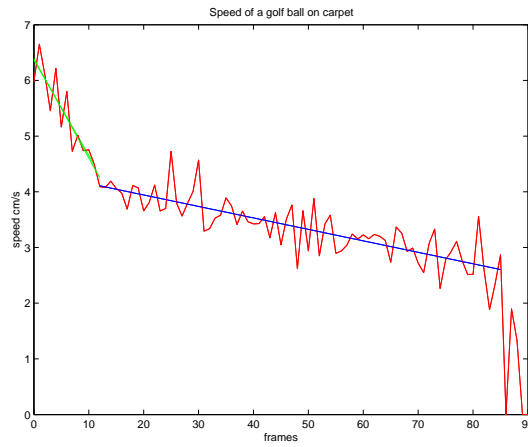


FIGURE 1. Speed of a golf ball in cm/s. The balls starts slipping on the carpet, then rolling. Two linear functions have been fitted to the data. At the end, the ball cannot “climb” over the carpet and suddenly stops moving.

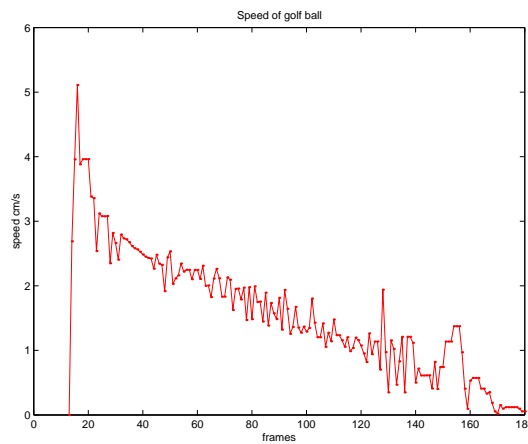


FIGURE 2. Speed of a golf ball in cm/s. The velocity curve in this example has the same structure as the previous one.

corresponds to  $5/7$  of the start velocity of 6 cm/s. When the velocity reaches a threshold of around 2 cm/s the ball suddenly “dies”.

Fi. 1 makes clear that there is a phase transition in the movement of the ball between the slipping and the rolling state. There is another phase transition when the ball velocity reaches the 2 cm/s threshold. In Fig. 1 two lines were fitted to the first two phases of the ball movement. This fit is better than a polynomial fit.

The best model for the ball, under the conditions we have been examining, is one which takes into account these two phase transitions. Within a given phase, the best model is a linear one.

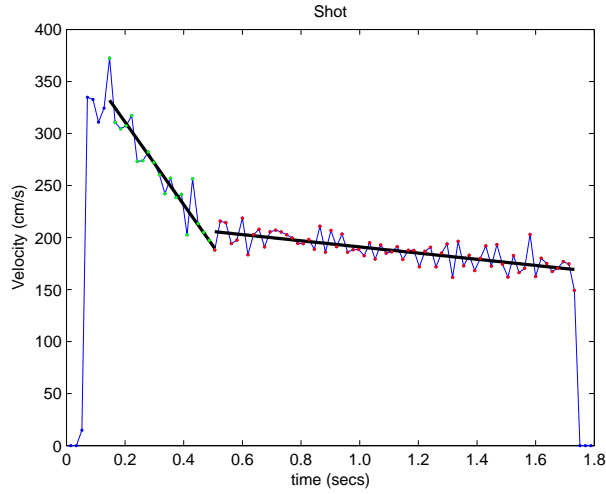


FIGURE 3. Optimal fit of two regression lines to the velocity data.

### 3. EXPERIMENTAL MEASUREMENTS

The analysis above is correct in terms of the two different phases that we can observe in the ball's movement. However, there is an experimental deviation from the factor predicted by the theory.

We captured 26 shots with our computer vision system. Fig. 3 shows one of them (velocity of the ball according to time). We tested systematically which subdivision of the data points could yield the best regression fit using two lines with different slopes. For the computation we just partitioned the data set into two subsets: data left or at the partition point, and data right of the partition point. The sum of minimal square errors for all data points relative to their assigned line is used as the cost function for the partition point. The point with minimal cost was selected by inspection. Fig. 3 shows one of those optimal fits.

Using this approach we found out that, as predicted by the theory, the transition from the gliding to the rolling phase occurs at a fixed factor of the initial velocity  $v_0$ . The factor differs from the theoretical factor and was found to be 0.5805 in our experiments. The standard deviation of the factor is low, just 0.042. The deceleration of our carpet on the ball during the gliding phase was found to be  $-3.47 \text{ m/s}^2$ , and  $-0.305 \text{ m/s}^2$  during the rolling phase. That corresponds to 35.4% of  $g$ , the earth acceleration, and 3.1% of  $g$ , respectively.

Using this insight and experimental values we then computed a prediction of the future ball movement as shown in Fig. ???. The prediction starts when the ball velocity exhibits a sudden jump. We just decelerate the measured velocity by  $0.354g$ . After ten frames we recompute the initial velocity  $v_0$ , which can be estimated better with the additional data, and make a new prediction, as shown by the lines in Fig. ???. We change the prediction from the gliding to the rolling phase when the predicted velocity breaks the threshold  $0.5805 \cdot v_0$ . After this phase transition the deceleration becomes  $0.305g$ .

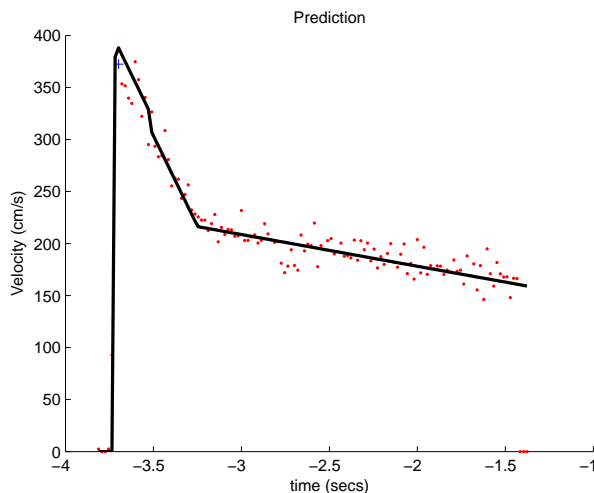


FIGURE 4. Prediction of the ball velocity. The prediction is given by the broken line, the actual measured velocity by the points in the graph. The first prediction is done right after the first frame. The corrected prediction trend is computed after ten frames. The slight displacement of the line with the higher negative slope is due to this correction.

The figure shows a very good correspondence of the prediction with the measured values.

Having a good prediction for the ball velocity it is then straightforward to program a good predictor for the ball position, just by integrating the predicted velocity with respect to time.

The factor 0.5805 is different from the factor  $5/7 = 0.71$ . The difference can be due to the moment of inertia assumed for the rolling ball. The golf balls used in our experiments do not have a constant density and their moment of inertia differs from  $\frac{2}{5}mR^2$ , the expression used in the calculations above.

#### 4. ENERGY DISSIPATED AS HEAT

Assume that a robot moving at velocity  $v$  is pushing a ball of mass  $m$  and radius  $R$ , which moves at the same velocity. If the ball is rolling without slipping on the surface, then the rolling rotational velocity  $\omega_r$  is related to  $v_r$  according to

$$v_r = \omega_r R$$

The energy of the ball at this velocity is given by the kinetic and rotational energy. The kinetic energy  $E_v$  is

$$E_v = \frac{1}{2}mv_r^2$$

The rotational energy  $E_{\omega_r}$  is given by

$$E_{\omega} = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_r^2 = \frac{1}{5}mv_r^2$$

This means that when the ball is rolling without slippage its total energy is

$$E_r = E_v + E_\omega = \frac{7}{10}mv_r^2$$

If the ball starts its movement slipping without rolling, with velocity  $v_0$ , its initial energy is

$$E_0 = \frac{1}{2}mv_0^2$$

When the ball starts rolling without slipping  $v_r = \frac{5}{7}v_0$ . The ball energy is therefore

$$E_r = \frac{7}{10}m\left(\frac{5}{7}v_0\right)^2 = \frac{5}{14}mv_0^2$$

The energy dissipated as heat is

$$E_0 - E_r = \frac{2}{14}mv_0^2$$

that is, almost  $2/7$  of the original energy.

## 5. DRIBBLING WITHOUT DRIBBLER

The analysis above explains why a robot pushing a ball at a constant velocity cannot lose the ball, if the robot constrains the rotational velocity of the ball.

If a robot is pushing a ball so fast that it does not rotate, whenever the ball loses contact with the robot, the ball decelerates because kinetic friction transfers linear to angular momentum. Even if robot and ball are moving originally with velocity  $v$ , the ball starts decelerating towards  $\frac{5}{7}v$ , while the robot can continue moving at the same speed. In other words: the ball cannot lose contact with the robot, because the robot immediately overtakes the ball.

Assume that the robot is pushing the ball which is rotating at an angular velocity  $\omega = \alpha\omega_r$ , lower than  $\omega_r$ , (that is  $\alpha < 1$ ). The term  $\omega_r$  is the non-slippage angular velocity. We know from the analysis above that if the robot lets the ball lose, the ball will decelerate until it rolls without slippage. The ball then moves with velocity  $v_r = R\omega_r$  and

$$m(v_r - v_0) = -\frac{2}{5}mR(\omega_r - \alpha\omega_r)$$

Therefore

$$v_r - v_0 = -\frac{2}{5}v_r(1 - \alpha)$$

and then

$$v_r = \frac{v_0}{\left(\frac{7}{5} - \frac{2}{5}\alpha\right)}$$

Fig 5 shows the final velocity of the ball when it starts rolling as percentage of the original velocity for different values of  $\alpha$ . When the ball, for example, is only rolling at  $\alpha = 0.5$  (half the angular velocity for rolling without slippage), and the robot loses the ball, then the ball decelerates to  $5/6$  of the original velocity.

It would be interesting to compute the time it takes the ball until it decelerates to rolling without slipping, but this computation depends on the coefficient of kinetic friction, which must be measured experimentally.

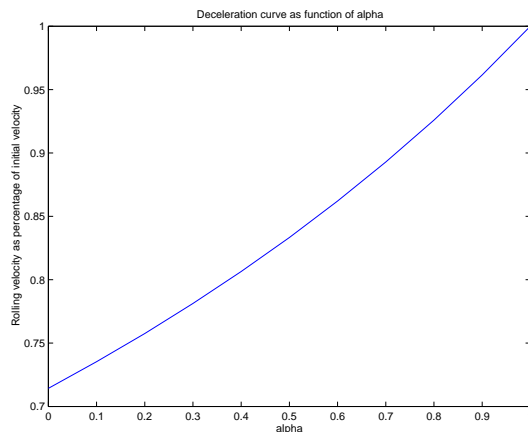


FIGURE 5. The curve shows the final rolling velocity as percentage of the original velocity, for different values of  $\alpha$ .

## 6. CONCLUSIONS

When the golf ball rolls on the RoboCup carpet, energy is lost due to the elastic properties of the carpet. The ball is rigid but not perfectly spherical, due to the golf ball dimples. This produces some additional loss of energy.

When the ball starts moving with velocity  $v_0$ , it slips on the surface until its forward velocity is  $\frac{5}{7}v_0$ . It then decelerates linearly until a threshold is reached and the ball suddenly stops.

The deceleration when the ball is rolling comes from “rolling friction”, very difficult to analyze since it depends on many factors. Apparently the deceleration is constant and does not depend, in some range, on the forward velocity of the ball.

This analysis should make it possible to program a good ball movement predictor. The data from the computer vision is rather noisy. It must be stabilized with a Kalman filter in order to get a better predictor.

There are only two constants that have to be determined empirically. The kinetic friction coefficient (which determines the slope of the first phase of the velocity curve) and the deceleration on a rolling ball (which depends on the specific type of carpet). There is a third constant which is not so important: the threshold of velocity under which the ball movement collapses. Since this value is rather low (under 1 cm/s) it does not affect the important calculations in real play.

Given the model it should be possible to compute all constants in the model automatically, collecting data and fitting a two segment linear function. Plug & Play.

## REFERENCES

1. Peter Maybeck, *Stochastic Models, Estimation and Control*, Academic Press. Inc, 1979. ent of Computer Science, University of North Carolina at Chapel Hill, updated in 2002.